Package: Rlinsolve (via r-universe)

September 7, 2024

Type Package

Title Iterative Solvers for (Sparse) Linear System of Equations

Version 0.3.3

Description Solving a system of linear equations is one of the most
fundamental computational problems for many fields of
mathematical studies, such as regression problems from
statistics or numerical partial differential equations. We
provide basic stationary iterative solvers such as Jacobi,
Gauss-Seidel, Successive Over-Relaxation and SSOR methods.
Nonstationary, also known as Krylov subspace methods are also
provided. Sparse matrix computation is also supported in that
solving large and sparse linear systems can be manageable using
'Matrix' package along with 'RcppArmadillo'. For a more
detailed description, see a book by Saad (2003)
<doi:10.1137/1.9780898718003>.

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Encoding UTF-8

Imports Rcpp (>= 0.12.4), Matrix, Rdpack, stats, utils

LinkingTo Rcpp, RcppArmadillo

RoxygenNote 7.2.3

RdMacros Rdpack

NeedsCompilation yes

Repository https://kisungyou.r-universe.dev

RemoteUrl https://github.com/kisungyou/rlinsolve

RemoteRef HEAD

RemoteSha fcec6e39249ff4239eec27e36262a66590f5b569

Contents

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aux.fisch   *Generate a 2-dimensional discrete Poisson matrix*

Description

Poisson equation is one of most well-known elliptic partial differential equations. In order to give a concrete example, a discrete Poisson matrix is generated, assuming we have N number of grid points for each dimension under square domain. *fisch* is a German word for Poisson.

Usage

```
aux.fisch(N, sparse = FALSE)
```

Arguments

N	the number of grid points for each direction.
sparse	a logical; TRUE for returning sparse matrix, FALSE otherwise.

Value

an $(N^2 \times N^2)$ matrix having block banded structure.

References

Golub, G. H. and Van Loan, C. F. (1996) *Matrix Computations, 3rd Ed.*, pages 177–180.

Examples

```
## generate dense and sparse Poisson matrix of size 25 by 25.
A = aux.fisch(5, sparse=FALSE)
B = aux.fisch(5, sparse=TRUE)
(all(A==B)) # TRUE if two matrices are equal.
```

---

lsolve.bicg | *Biconjugate Gradient method*

Description

Biconjugate Gradient(BiCG) method is a modification of Conjugate Gradient for nonsymmetric systems using evaluations with respect to A^T as well as A in matrix-vector multiplications. For an overdetermined system where `nrow(A)>ncol(A)`, it is automatically transformed to the normal equation. Underdetermined system - `nrow(A)<ncol(A)` - is not supported. Preconditioning matrix M, in theory, should be symmetric and positive definite with fast computability for inverse, though it is not limited until the solver level.

Usage

```
lsolve.bicg(
  A,
  B,
  xinit = NA,
  reltol = 1e-05,
  maxiter = 10000,
  preconditioner = diag(ncol(A)),
  verbose = TRUE
)
```

Arguments

A	an $(m \times n)$ dense or sparse matrix. See also `sparseMatrix`.
B	a vector of length m or an $(m \times k)$ matrix (dense or sparse) for solving k systems simultaneously.
xinit	a length-n vector for initial starting point. NA to start from a random initial point near 0.
reltol	tolerance level for stopping iterations.
maxiter	maximum number of iterations allowed.
preconditioner	an $(n \times n)$ preconditioning matrix; default is an identity matrix.
verbose	a logical; TRUE to show progress of computation.

Value

a named list containing

x solution; a vector of length n or a matrix of size $(n \times k)$.

iter the number of iterations required.

errors a vector of errors for stopping criterion.

References

Fletcher R (1976). "Conjugate gradient methods for indefinite systems." In Watson GA (ed.), *Numerical Analysis*, volume 506, 73–89. Springer Berlin Heidelberg, Berlin, Heidelberg. ISBN 978-3-540-07610-0 978-3-540-38129-7.

Voevodin VV (1983). "The question of non-self-adjoint extension of the conjugate gradients method is closed." *USSR Computational Mathematics and Mathematical Physics*, **23**(2), 143–144. ISSN 00415553.

Examples

```
## Overdetermined System
set.seed(100)
A = matrix(rnorm(10*5),nrow=10)
x = rnorm(5)
b = A%*%x

out1 = lsolve.cg(A,b)
out2 = lsolve.bicg(A,b)
matout = cbind(matrix(x),out1$x, out2$x);
colnames(matout) = c("true x","CG result", "BiCG result")
print(matout)
```

---

lsolve.bicgstab   *Biconjugate Gradient Stabilized Method*

Description

Biconjugate Gradient Stabilized(BiCGSTAB) method is a stabilized version of Biconjugate Gradient method for nonsymmetric systems using evaluations with respect to A^T as well as A in matrix-vector multiplications. For an overdetermined system where nrow(A)>ncol(A), it is automatically transformed to the normal equation. Underdetermined system - nrow(A)<ncol(A) - is not supported. Preconditioning matrix M, in theory, should be symmetric and positive definite with fast computability for inverse, though it is not limited until the solver level.

Usage

```
lsolve.bicgstab(
  A,
  B,
  xinit = NA,
  reltol = 1e-05,
  maxiter = 1000,
  preconditioner = diag(ncol(A)),
  verbose = TRUE
)
```

Arguments

A	an $(m \times n)$ dense or sparse matrix. See also `sparseMatrix`.
B	a vector of length m or an $(m \times k)$ matrix (dense or sparse) for solving k systems simultaneously.
xinit	a length-n vector for initial starting point. NA to start from a random initial point near 0.
reltol	tolerance level for stopping iterations.
maxiter	maximum number of iterations allowed.
preconditioner	an $(n \times n)$ preconditioning matrix; default is an identity matrix.
verbose	a logical; `TRUE` to show progress of computation.

Value

a named list containing

x solution; a vector of length n or a matrix of size $(n \times k)$.

iter the number of iterations required.

errors a vector of errors for stopping criterion.

References

van der Vorst HA (1992). "Bi-CGSTAB: A Fast and Smoothly Converging Variant of Bi-CG for the Solution of Nonsymmetric Linear Systems." *SIAM Journal on Scientific and Statistical Computing*, **13**(2), 631–644. ISSN 0196-5204, 2168-3417.

Examples

```
## Overdetermined System
set.seed(100)
A = matrix(rnorm(10*5),nrow=10)
x = rnorm(5)
b = A%*%x

out1 = lsolve.cg(A,b)
out2 = lsolve.bicg(A,b)
out3 = lsolve.bicgstab(A,b)
matout = cbind(matrix(x),out1$x, out2$x, out3$x);
colnames(matout) = c("true x","CG result", "BiCG result", "BiCGSTAB result")
print(matout)
```

---

lsolve.cg   *Conjugate Gradient method*

Description

Conjugate Gradient(CG) method is an iterative algorithm for solving a system of linear equations where the system is symmetric and positive definite. For a square matrix A, it is required to be symmetric and positive definite. For an overdetermined system where `nrow(A)>ncol(A)`, it is automatically transformed to the normal equation. Underdetermined system - `nrow(A)<ncol(A)` - is not supported. Preconditioning matrix M, in theory, should be symmetric and positive definite with fast computability for inverse, though it is not limited until the solver level.

Usage

```
lsolve.cg(
  A,
  B,
  xinit = NA,
  reltol = 1e-05,
  maxiter = 10000,
  preconditioner = diag(ncol(A)),
  adjsym = TRUE,
  verbose = TRUE
)
```

Arguments

A	an $(m \times n)$ dense or sparse matrix. See also [sparseMatrix]().
B	a vector of length m or an $(m \times k)$ matrix (dense or sparse) for solving k systems simultaneously.
xinit	a length-n vector for initial starting point. NA to start from a random initial point near 0.
reltol	tolerance level for stopping iterations.
maxiter	maximum number of iterations allowed.
preconditioner	an $(n \times n)$ preconditioning matrix; default is an identity matrix.
adjsym	a logical; TRUE to symmetrize the system by transforming the system into normal equation, FALSE otherwise.
verbose	a logical; TRUE to show progress of computation.

Value

a named list containing

x solution; a vector of length n or a matrix of size $(n \times k)$.

iter the number of iterations required.

errors a vector of errors for stopping criterion.

References

Hestenes MR, Stiefel E (1952). "Methods of conjugate gradients for solving linear systems." *Journal of Research of the National Bureau of Standards*, **49**(6), 409. ISSN 0091-0635.

Examples

```
## Overdetermined System
set.seed(100)
A = matrix(rnorm(10*5),nrow=10)
x = rnorm(5)
b = A%*%x

out1 = lsolve.sor(A,b,w=0.5)
out2 = lsolve.cg(A,b)
matout = cbind(matrix(x),out1$x, out2$x);
colnames(matout) = c("true x","SSOR result", "CG result")
print(matout)
```

---

lsolve.cgs   *Conjugate Gradient Squared method*

Description

Conjugate Gradient Squared(CGS) method is an extension of Conjugate Gradient method where the system is symmetric and positive definite. It aims at achieving faster convergence using an idea of contraction operator twice. For a square matrix A,it is required to be symmetric and positive definite. For an overdetermined system where nrow(A)>ncol(A), it is automatically transformed to the normal equation. Underdetermined system - nrow(A)<ncol(A) - is not supported. Preconditioning matrix M, in theory, should be symmetric and positive definite with fast computability for inverse, though it is not limited until the solver level.

Usage

```
lsolve.cgs(
  A,
  B,
  xinit = NA,
  reltol = 1e-05,
  maxiter = 10000,
  preconditioner = diag(ncol(A)),
  adjsym = TRUE,
  verbose = TRUE
)
```

Arguments

A	an $(m \times n)$ dense or sparse matrix. See also [sparseMatrix]().
B	a vector of length m or an $(m \times k)$ matrix (dense or sparse) for solving k systems simultaneously.
xinit	a length-n vector for initial starting point. NA to start from a random initial point near 0.
reltol	tolerance level for stopping iterations.
maxiter	maximum number of iterations allowed.
preconditioner	an $(n \times n)$ preconditioning matrix; default is an identity matrix.
adjsym	a logical; TRUE to symmetrize the system by transforming the system into normal equation, FALSE otherwise.
verbose	a logical; TRUE to show progress of computation.

Value

a named list containing

x solution; a vector of length n or a matrix of size $(n \times k)$.

iter the number of iterations required.

errors a vector of errors for stopping criterion.

References

Sonneveld P (1989). "CGS, A Fast Lanczos-Type Solver for Nonsymmetric Linear systems." *SIAM Journal on Scientific and Statistical Computing*, **10**(1), 36–52. ISSN 0196-5204, 2168-3417.

Examples

```
## Overdetermined System
set.seed(100)
A = matrix(rnorm(10*5),nrow=10)
x = rnorm(5)
b = A%*%x

out1 = lsolve.cg(A,b)
out2 = lsolve.cgs(A,b)
matout = cbind(matrix(x),out1$x, out2$x);
colnames(matout) = c("true x","CG result", "CGS result")
print(matout)
```

lsolve.cheby   *Chebyshev Method*

Description

Chebyshev method - also known as Chebyshev iteration - avoids computation of inner product, enabling distributed-memory computation to be more efficient at the cost of requiring a priori knowledge on the range of spectrum for matrix A.

Usage

```
lsolve.cheby(
  A,
  B,
  xinit = NA,
  reltol = 1e-05,
  maxiter = 10000,
  preconditioner = diag(ncol(A)),
  adjsym = TRUE,
  verbose = TRUE
)
```

Arguments

A	an $(m \times n)$ dense or sparse matrix. See also `sparseMatrix`.
B	a vector of length m or an $(m \times k)$ matrix (dense or sparse) for solving k systems simultaneously.
xinit	a length-n vector for initial starting point. NA to start from a random initial point near 0.
reltol	tolerance level for stopping iterations.
maxiter	maximum number of iterations allowed.
preconditioner	an $(n \times n)$ preconditioning matrix; default is an identity matrix.
adjsym	a logical; TRUE to symmetrize the system by transforming the system into normal equation, FALSE otherwise.
verbose	a logical; TRUE to show progress of computation.

Value

a named list containing

x solution; a vector of length n or a matrix of size $(n \times k)$.

iter the number of iterations required.

errors a vector of errors for stopping criterion.

References

Gutknecht MH, Röllin S (2002). "The Chebyshev iteration revisited." *Parallel Computing*, **28**(2), 263–283. ISSN 01678191.

Examples

```
## Overdetermined System
set.seed(100)
A = matrix(rnorm(10*5),nrow=10)
x = rnorm(5)
b = A%*%x

out1 = lsolve.sor(A,b,w=0.5)
out2 = lsolve.cheby(A,b)
matout = cbind(x, out1$x, out2$x);
colnames(matout) = c("original x","SOR result", "Chebyshev result")
print(matout)
```

---

lsolve.gmres   *Generalized Minimal Residual method*

Description

GMRES is a generic iterative solver for a nonsymmetric system of linear equations. As its name suggests, it approximates the solution using Krylov vectors with minimal residuals.

Usage

```
lsolve.gmres(
  A,
  B,
  xinit = NA,
  reltol = 1e-05,
  maxiter = 1000,
  preconditioner = diag(ncol(A)),
  restart = (ncol(A) - 1),
  verbose = TRUE
)
```

Arguments

A	an $(m \times n)$ dense or sparse matrix. See also sparseMatrix.
B	a vector of length m or an $(m \times k)$ matrix (dense or sparse) for solving k systems simultaneously.
xinit	a length-n vector for initial starting point. NA to start from a random initial point near 0.

reltol	tolerance level for stopping iterations.
maxiter	maximum number of iterations allowed.
preconditioner	an $(n \times n)$ preconditioning matrix; default is an identity matrix.
restart	the number of iterations before restart.
verbose	a logical; TRUE to show progress of computation.

Value

a named list containing

x solution; a vector of length n or a matrix of size $(n \times k)$.

iter the number of iterations required.

errors a vector of errors for stopping criterion.

References

Saad Y, Schultz MH (1986). "GMRES: A Generalized Minimal Residual Algorithm for Solving Nonsymmetric Linear Systems." *SIAM Journal on Scientific and Statistical Computing*, **7**(3), 856–869. ISSN 0196-5204, 2168-3417.

Examples

```
## Overdetermined System
set.seed(100)
A = matrix(rnorm(10*5),nrow=10)
x = rnorm(5)
b = A%*%x

out1 = lsolve.cg(A,b)
out3_1 = lsolve.gmres(A,b,restart=2)
out3_2 = lsolve.gmres(A,b,restart=3)
out3_3 = lsolve.gmres(A,b,restart=4)
matout = cbind(matrix(x),out1$x, out3_1$x, out3_2$x, out3_3$x);
colnames(matout) = c("true x","CG", "GMRES(2)", "GMRES(3)", "GMRES(4)")
print(matout)
```

---

lsolve.gs *Gauss-Seidel method*

Description

Gauss-Seidel(GS) method is an iterative algorithm for solving a system of linear equations, with a decomposition $A = D + L + U$ where D is a diagonal matrix and L and U are strictly lower/upper triangular matrix respectively. For a square matrix A, it is required to be diagonally dominant or symmetric and positive definite. For an overdetermined system where nrow(A)>ncol(A), it is automatically transformed to the normal equation. Underdetermined system - nrow(A)<ncol(A) - is not supported.

Usage

```
lsolve.gs(
  A,
  B,
  xinit = NA,
  reltol = 1e-05,
  maxiter = 1000,
  adjsym = TRUE,
  verbose = TRUE
)
```

Arguments

A	an $(m \times n)$ dense or sparse matrix. See also [sparseMatrix]().
B	a vector of length m or an $(m \times k)$ matrix (dense or sparse) for solving k systems simultaneously.
xinit	a length-n vector for initial starting point. NA to start from a random initial point near 0.
reltol	tolerance level for stopping iterations.
maxiter	maximum number of iterations allowed.
adjsym	a logical; TRUE to symmetrize the system by transforming the system into normal equation, FALSE otherwise.
verbose	a logical; TRUE to show progress of computation.

Value

a named list containing

x solution; a vector of length n or a matrix of size $(n \times k)$.

iter the number of iterations required.

errors a vector of errors for stopping criterion.

References

Demmel JW (1997). *Applied Numerical Linear Algebra*. Society for Industrial and Applied Mathematics. ISBN 978-0-89871-389-3 978-1-61197-144-6.

Examples

```
## Overdetermined System
set.seed(100)
A = matrix(rnorm(10*5),nrow=10)
x = rnorm(5)
b = A%*%x

out = lsolve.gs(A,b)
matout = cbind(matrix(x),out$x); colnames(matout) = c("true x","est from GS")
```

```
print(matout)
```

`lsolve.jacobi`   *Jacobi method*

Description

Jacobi method is an iterative algorithm for solving a system of linear equations, with a decomposition $A = D + R$ where D is a diagonal matrix. For a square matrix A, it is required to be diagonally dominant. For an overdetermined system where `nrow(A)>ncol(A)`, it is automatically transformed to the normal equation. Underdetermined system - `nrow(A)<ncol(A)` - is not supported.

Usage

```
lsolve.jacobi(
  A,
  B,
  xinit = NA,
  reltol = 1e-05,
  maxiter = 1000,
  weight = 2/3,
  adjsym = TRUE,
  verbose = TRUE
)
```

Arguments

A	an $(m \times n)$ dense or sparse matrix. See also [sparseMatrix](#).
B	a vector of length m or an $(m \times k)$ matrix (dense or sparse) for solving k systems simultaneously.
xinit	a length-n vector for initial starting point. NA to start from a random initial point near 0.
reltol	tolerance level for stopping iterations.
maxiter	maximum number of iterations allowed.
weight	a real number in $(0, 1]$; 1 for native Jacobi.
adjsym	a logical; TRUE to symmetrize the system by transforming the system into normal equation, FALSE otherwise.
verbose	a logical; TRUE to show progress of computation.

Value

a named list containing

x solution; a vector of length n or a matrix of size $(n \times k)$.

iter the number of iterations required.

errors a vector of errors for stopping criterion.

References

Demmel JW (1997). *Applied Numerical Linear Algebra*. Society for Industrial and Applied Mathematics. ISBN 978-0-89871-389-3 978-1-61197-144-6.

Examples

```
## Overdetermined System
set.seed(100)
A = matrix(rnorm(10*5),nrow=10)
x = rnorm(5)
b = A%*%x

out1 = lsolve.jacobi(A,b,weight=1,verbose=FALSE)   # unweighted
out2 = lsolve.jacobi(A,b,verbose=FALSE)            # weight of 0.66
out3 = lsolve.jacobi(A,b,weight=0.5,verbose=FALSE) # weight of 0.50
print("* lsolve.jacobi : overdetermined case example")
print(paste("*   error for unweighted    Jacobi case : ",norm(out1$x-x)))
print(paste("*   error for 0.66 weighted Jacobi case : ",norm(out2$x-x)))
print(paste("*   error for 0.50 weighted Jacobi case : ",norm(out3$x-x)))
```

---

lsolve.qmr   *Quasi Minimal Residual Method*

Description

Quasia-Minimal Resudial(QMR) method is another remedy of the BiCG which shows rather irregular convergence behavior. It adapts to solve the reduced tridiagonal system in a least squares sense and its convergence is known to be quite smoother than BiCG.

Usage

```
lsolve.qmr(
  A,
  B,
  xinit = NA,
  reltol = 1e-05,
  maxiter = 1000,
  preconditioner = diag(ncol(A)),
  verbose = TRUE
)
```

Arguments

A	an $(m \times n)$ dense or sparse matrix. See also [sparseMatrix](#).
B	a vector of length m or an $(m \times k)$ matrix (dense or sparse) for solving k systems simultaneously.

xinit	a length-n vector for initial starting point. NA to start from a random initial point near 0.
reltol	tolerance level for stopping iterations.
maxiter	maximum number of iterations allowed.
preconditioner	an $(n \times n)$ preconditioning matrix; default is an identity matrix.
verbose	a logical; TRUE to show progress of computation.

Value

a named list containing

x solution; a vector of length n or a matrix of size $(n \times k)$.

iter the number of iterations required.

errors a vector of errors for stopping criterion.

References

Freund RW, Nachtigal NM (1991). "QMR: a quasi-minimal residual method for non-Hermitian linear systems." *Numerische Mathematik*, **60**(1), 315–339. ISSN 0029-599X, 0945-3245.

Examples

```
## Overdetermined System
set.seed(100)
A = matrix(rnorm(10*5),nrow=10)
x = rnorm(5)
b = A%*%x

out1 = lsolve.cg(A,b)
out2 = lsolve.bicg(A,b)
out3 = lsolve.qmr(A,b)
matout = cbind(matrix(x),out1$x, out2$x, out3$x);
colnames(matout) = c("true x","CG result", "BiCG result", "QMR result")
print(matout)
```

---

lsolve.sor *Successive Over-Relaxation method*

Description

Successive Over-Relaxation(SOR) method is a variant of Gauss-Seidel method for solving a system of linear equations, with a decomposition $A = D + L + U$ where D is a diagonal matrix and L and U are strictly lower/upper triangular matrix respectively. For a square matrix A, it is required to be diagonally dominant or symmetric and positive definite like GS method. For an overdetermined system where nrow(A)>ncol(A), it is automatically transformed to the normal equation. Underdetermined system - nrow(A)<ncol(A) - is not supported.

Usage

```
lsolve.sor(
  A,
  B,
  xinit = NA,
  reltol = 1e-05,
  maxiter = 1000,
  w = 1,
  adjsym = TRUE,
  verbose = TRUE
)
```

Arguments

A	an $(m \times n)$ dense or sparse matrix. See also `sparseMatrix`.
B	a vector of length m or an $(m \times k)$ matrix (dense or sparse) for solving k systems simultaneously.
xinit	a length-n vector for initial starting point. NA to start from a random initial point near 0.
reltol	tolerance level for stopping iterations.
maxiter	maximum number of iterations allowed.
w	a weight value in $(0, 2)$.; w=1 leads to Gauss-Seidel method.
adjsym	a logical; TRUE to symmetrize the system by transforming the system into normal equation, FALSE otherwise.
verbose	a logical; TRUE to show progress of computation.

Value

a named list containing

x solution; a vector of length n or a matrix of size $(n \times k)$.

iter the number of iterations required.

errors a vector of errors for stopping criterion.

References

Demmel JW (1997). *Applied Numerical Linear Algebra*. Society for Industrial and Applied Mathematics. ISBN 978-0-89871-389-3 978-1-61197-144-6.

Examples

```
## Overdetermined System
set.seed(100)
A = matrix(rnorm(10*5),nrow=10)
x = rnorm(5)
b = A%*%x
```

```
out1 = lsolve.sor(A,b)
out2 = lsolve.sor(A,b,w=0.5)
out3 = lsolve.sor(A,b,w=1.5)
matout = cbind(matrix(x),out1$x, out2$x, out3$x);
colnames(matout) = c("true x","SOR 1 = GS", "SOR w=0.5", "SOR w=1.5")
print(matout)
```

---

lsolve.ssor   *Symmetric Successive Over-Relaxation method*

Description

Symmetric Successive Over-Relaxation(SSOR) method is a variant of Gauss-Seidel method for solving a system of linear equations, with a decomposition $A = D + L + U$ where D is a diagonal matrix and L and U are strictly lower/upper triangular matrix respectively. For a square matrix A, it is required to be diagonally dominant or symmetric and positive definite like GS method. For an overdetermined system where nrow(A)>ncol(A), it is automatically transformed to the normal equation. Underdetermined system - nrow(A)<ncol(A) - is not supported.

Usage

```
lsolve.ssor(
  A,
  B,
  xinit = NA,
  reltol = 1e-05,
  maxiter = 1000,
  w = 1,
  adjsym = TRUE,
  verbose = TRUE
)
```

Arguments

A	an $(m \times n)$ dense or sparse matrix. See also [sparseMatrix]().
B	a vector of length m or an $(m \times k)$ matrix (dense or sparse) for solving k systems simultaneously.
xinit	a length-n vector for initial starting point. NA to start from a random initial point near 0.
reltol	tolerance level for stopping iterations.
maxiter	maximum number of iterations allowed.
w	a weight value in $(0, 2)$.; w=1 leads to Gauss-Seidel method.
adjsym	a logical; TRUE to symmetrize the system by transforming the system into normal equation, FALSE otherwise.
verbose	a logical; TRUE to show progress of computation.

Value

a named list containing

x solution; a vector of length n or a matrix of size $(n \times k)$.

iter the number of iterations required.

errors a vector of errors for stopping criterion.

References

Demmel JW (1997). *Applied Numerical Linear Algebra*. Society for Industrial and Applied Mathematics. ISBN 978-0-89871-389-3 978-1-61197-144-6.

Examples

```
## Overdetermined System
set.seed(100)
A = matrix(rnorm(10*5),nrow=10)
x = rnorm(5)
b = A%*%x

out1 = lsolve.ssor(A,b)
out2 = lsolve.ssor(A,b,w=0.5)
out3 = lsolve.ssor(A,b,w=1.5)
matout = cbind(matrix(x),out1$x, out2$x, out3$x);
colnames(matout) = c("true x","SSOR w=1", "SSOR w=0.5", "SSOR w=1.5")
print(matout)
```

Rlinsolve   *A Collection of Iterative Solvers for (Sparse) Linear System of Equations*

Description

Solving a system of linear equations is one of the most fundamental computational problems for many fields of mathematical studies, such as regression from statistics or numerical partial differential equations. We provide a list of both stationary and nonstationary solvers. Sparse matrix class from **Matrix** is also supported for large sparse system.

Non-square matrix

For a matrix A of size (m-by-n), we say the system is **overdetermined** if m>n, **underdetermined** if m<n, or **squared** if m=n. In the current version, underdetermined system is not supported; it will later appear with sparse constraints. For an overdetermined system, it automatically transforms the problem into *normal equation*, i.e.,

$$Ax = b \rightarrow A^T A x = A^T b$$

even though if suffers from worse condition number while having desirable property of a system to be symmetric and positive definite.

Sparsity

RcppArmadillo is extensively used in the package. In order for bullet-proof transition between dense and sparse matrix, only 3 of 12 RcppArmadillo-supported sparse matrix formats have access to our algorithms; "dgCMatrix","dtCMatrix" and "dsCMatrix". Please see the vignette on sparse matrix support from RcppArmadillo. If either of two inputs A or b is sparse, all matrices involved are automatically transformed into sparse matrices.

Composition of the Package

Following is a list of stationary methods,

lsolve.jacobi Jacobi method

lsolve.gs Gauss-Seidel method

lsolve.sor Successive Over-Relaxation method

lsolve.ssor Symmetric Successive Over-Relaxation method

as well as nonstationary (or, Krylov subspace) methods,

lsolve.bicg Bi-Conjugate Gradient method

lsolve.bicgstab Bi-Conjugate Gradient Stabilized method

lsolve.cg Conjugate Gradient method

lsolve.cgs Conjugate Gradient Squared method

lsolve.cheby Chebyshev method

lsolve.gmres Generalized Minimal Residual method

lsolve.qmr Quasi-Minimal Residual method

Also, aux.fisch is provided to generate a sparse system of discrete Poisson matrix from finite difference approximation scheme of Poisson equation on 2-dimensional square domain.

References

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